

## PACE INSTITUTE OF TECHNOLOGY \& SCIENCES::ONGOLE (AUTONOMOUS)

## I B.TECH I SEMESTER END SUPPLEMENTARY EXAMINATIONS, FEB - 2023 MATHEMATICS-I

(Common to All Branches)
Time: 3 hours
Max. Marks: 60

## Note: Question Paper consists of Two parts (Part-A and Part-B) <br> PART-A

Answer all the questions in Part-A $(5 X 2=10 \mathrm{M})$

| Q.No. |  | Questions | Marks | CO | KL |
| :---: | :---: | :--- | :---: | :---: | :---: |
| 1. | a) | Find the orthogonal trajectories of the family of curves $x^{2}+y^{2}=a^{2}$. | $[2 \mathrm{M}]$ | 1 | 2 |
|  | b) | Write the application of linear differential equation of second and higher <br> order. | $[2 \mathrm{M}]$ | 2 | 1 |
|  | c) | Find the Laplace Transform of e ${ }^{3 \mathrm{t}}+9$. | $[2 \mathrm{M}]$ | 3 | 2 |
|  | d) | Evaluate $L^{-1}\left[\frac{1}{s(s-2)}\right]$. | $[2 \mathrm{M}]$ | 4 | 2 |
|  | e) | State Taylor's series expansion of functions of two variables. | $[2 \mathrm{M}]$ | 5 | 1 |

## PART-B

Answer One Question from each UNIT (5X10=50M)

| Q.No. |  | Questions | Marks | CO | KL |
| :---: | :---: | :---: | :---: | :---: | :---: |
| UNIT-I |  |  |  |  |  |
| 2. | a) | Solve the differential equation of $\left(1-x^{2}\right) \frac{d y}{d x}+x y=y^{3} \sin ^{-1} x$. | [5M] | 1 | 3 |
|  | b) | Find the orthogonal trajectories of the family of cardioids $r=a(1-\cos \theta)$ where $a$ is the parameter. | [5M] | 1 | 2 |
| OR |  |  |  |  |  |
| 3. | a) | Solve the differential equation $x^{2} y d x-\left(x^{3}+y^{3}\right) d y=0$ | [5M] | 1 | 3 |
|  | b) | Uranium disintegrates at a rate proportional to the amount present at any instant. If $\mathrm{M}_{1}$ and ${ }^{1}{ }_{2} M$ grams of uranium that are present at times T and $\mathrm{T} \quad 2$ respectively. Show that the half-life of uranium is $\mathrm{T}_{2}-\mathrm{T}_{1}$. | [5M] | 1 | 3 |
| UNIT-II |  |  |  |  |  |
| 4. | a) | Solve the differential equation $\left(D^{2}+5 D+4\right) y=2 \sin a x$. | [5M] | 2 | 3 |
|  | b) | Solve the differential equation $\left(D^{2}+1\right) y=\cos x$ by the method of variation of parameters. | [5M] | 2 | 3 |
| OR |  |  |  |  |  |
| 5. | a) | Solve the differential equation $\left(D^{2}+4\right) y=e^{x} \sin ^{2} x$. | [5M] | 2 | 3 |
|  | b) | Solve the differential equation $\left(D^{2}+D+1\right) y=x^{3}$. | [5M] | 2 | 3 |
| UNIT-III |  |  |  |  |  |
| 6. | a) | Find $L_{\{ }\{f(t)\}$ where $f(t)=\left\{\begin{array}{c}\cos \left(t-\frac{2 \pi}{3}\right) \text { if } t>\frac{2 \pi}{3} \\ 0 \text { if } t<\frac{2 \pi}{3}\end{array}\right.$. | [5M] | 3 | 2 |
|  | b) | Find the Laplace Transform of $f(t)=e^{3 t} \operatorname{Sin}^{2} t$. | [5M] | 3 | 2 |
| OR |  |  |  |  |  |


| 7. |  | By using the expansion of $\sin \mathrm{x}$ show that $L(\sin \sqrt{t})=\frac{\sqrt{\pi}}{2 s^{3 / 2}} e^{\frac{-1}{4 s}}$. | [10M] | 3 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| UNIT-IV |  |  |  |  |  |
| 8. | a) | Evaluate $L^{-1}\left[\frac{1+e^{-\pi s}}{s^{2}+1}\right]$. | [5M] | 4 | 2 |
|  | b) | Evaluate $L^{-1}\left[\frac{1}{s(s+1)^{3}}\right]$. | [5M] | 4 | 2 |
| OR |  |  |  |  |  |
| 9. |  | Solve the differential equation $\left(D^{2}+3 D+2\right) y=e^{-t}, y(0)=0, y^{\prime}(0)=1$ using Laplace transform | [10M] | 4 | 3 |
| UNIT-V |  |  |  |  |  |
| 10. | a) | Verify Euler's theorem for the function $u=\sin ^{-1} \frac{x}{y}+\tan ^{-1} \frac{y}{x}$ | [5M] | 5 | 3 |
|  | b) | If $u=x^{2}-y^{2}, v=2 x y$ where $x=r \cos \alpha, y=r \sin \alpha$ then show that $\frac{\partial(u, v)}{\partial(r, \alpha)}=4 r^{3}$. | [5M] | 5 | 2 |
| OR |  |  |  |  |  |
| 11. |  | Using Taylor's theorem to expand $f(x, y)=x^{2}+x y+y^{2}$ in powers of $x-1$ and $y-2$. | [10M] | 5 | 3 |

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